Infectious Disease Models 5: Intervention Impact on an Open Population

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CMPT 858

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Recall: The Importance of Susceptible Fraction

- Recall: Total # of susceptibles infected per unit time = # of Susceptibles * “Likelihood” a given susceptible will be infected per unit time = S*(“*Force of Infection*”) = $S(c(I/N)\beta)$
- The above can also be phrased as the following: $S(c(I/N)\beta)=I(c(S/N)\beta)$=# of Infectives * *Average # susceptibles infected per unit time by each infective*
- This implies that as Fraction of susceptibles falls=>Fraction of susceptibles surrounding each infective falls=>the rate of new infections falls (“Less fuel for the fire” leads to a smaller burning rate)

A Critical Throttle on Infection Spread: Fraction Susceptible (*f*)

- The fraction susceptible (here, S/N) is a key quantity limiting the spread of infection in a population
  - Recognizing its importance, we give this name *f* to the fraction of the population that issusceptible
- If contact patterns & infection duration remain unchanged and, then mean # of individuals infected by an infective over the course of their infection is $f*R_0$

Critical Immunization Threshold

- Consider an index infective arriving in a "worst case" scenario when noone else in the population is infective or recovered from the illness
  - In this case, that infective is most "efficient" in spreading
- The goal of vaccination is keep the fraction susceptible low enough that infection cannot establish itself even in this worst case
  - We do this by administering vaccines that makes a person (often temporarily) immune to infection
- We say that a population whose f is low enough that it is resistant to establishment of infection exhibits "herd immunity"

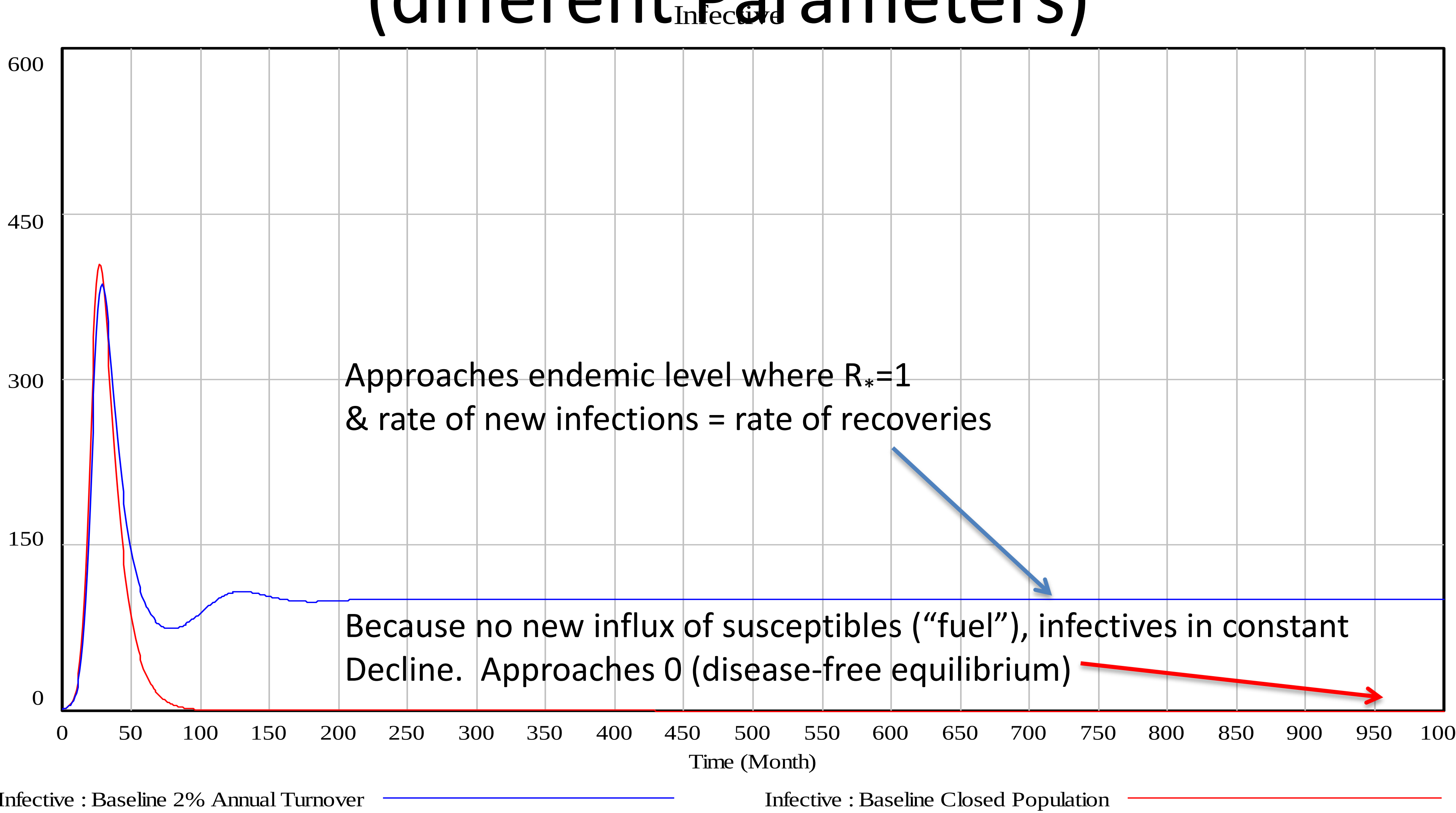
Critical Immunization Threshold

- Vaccination seeks to lower *f* such that f*R_0<1
- *Worst case: Suppose we have a population that is divided into immunized (vaccinated) and susceptible*
  - Let q_c be the critical fraction immunized to stop infection
  - *Then* $f=1-q_c$, $f*R_0<1 \Rightarrow (1-q_c)*R_0<1 \Rightarrow q_c>1-(1/R_0)$
- So if R_0 = 4 (as in our example), $q_c=0.75$ *(i.e. 75% of population must be immunized – just as we saw!)*

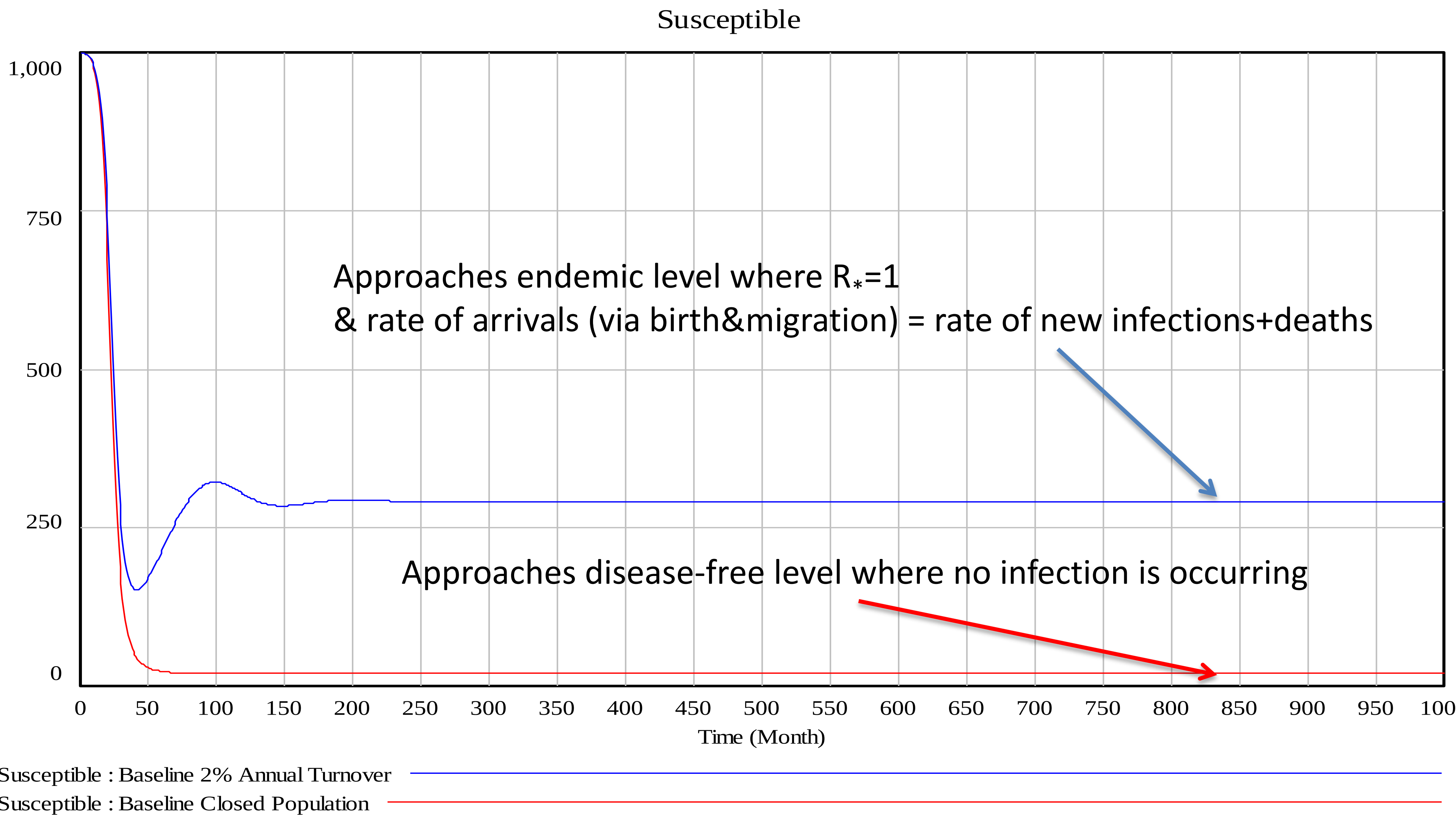
Open/Closed Population

	Case	Epidemic Occurs?	Steady-state	
			Fraction infective	Fraction susceptible
Open Population	$R_0>1$	Yes	Such that Infection rate=Recovery rate	$1/R_0$
	$R_0<1$	No	0	1
Closed Population	$R_0>1$	Yes	0	1
	$R_0<1$	No	0	1

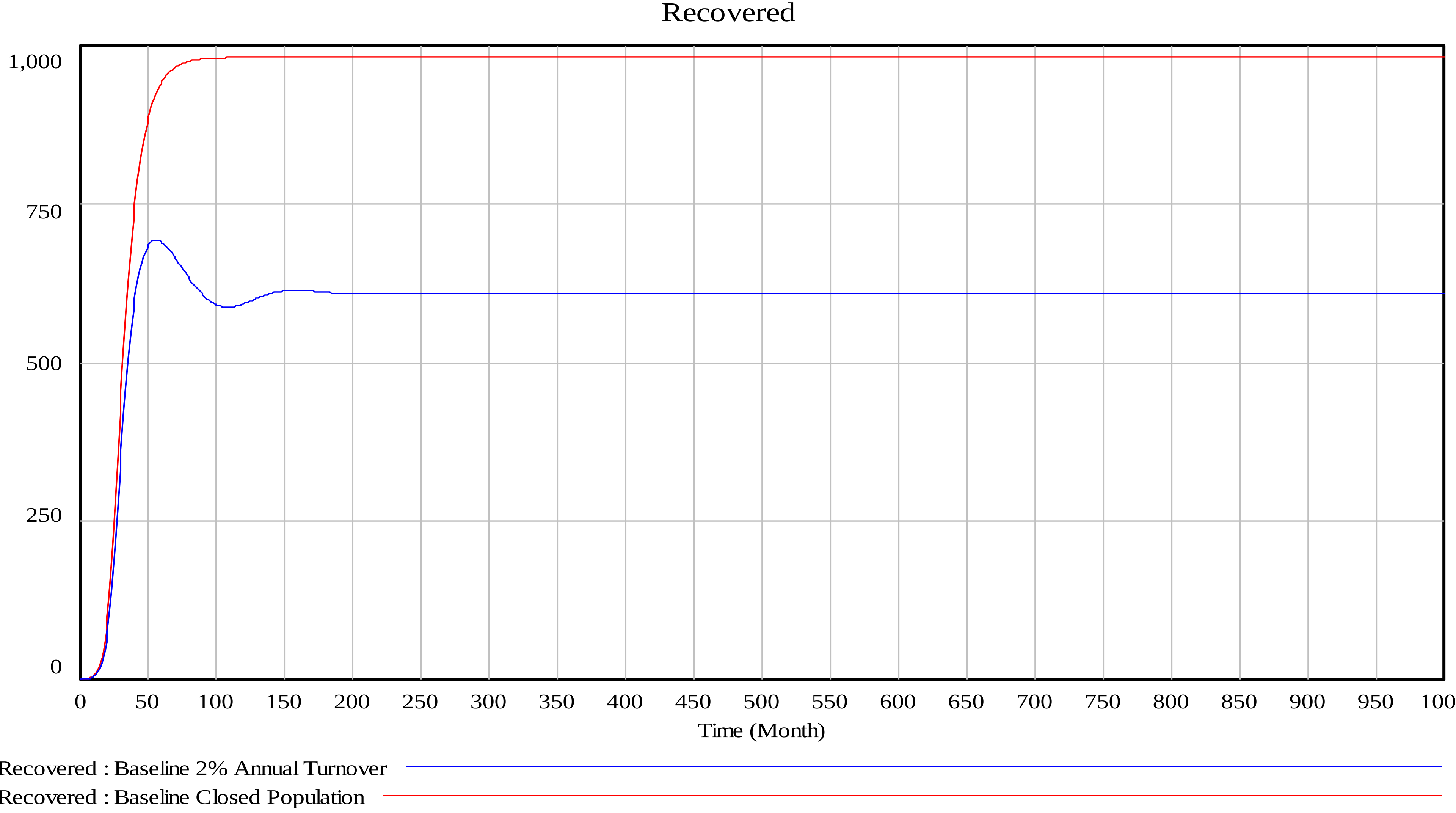
Effects of An Open Population (different Parameters)



Effects of An Open Population



Recovereds

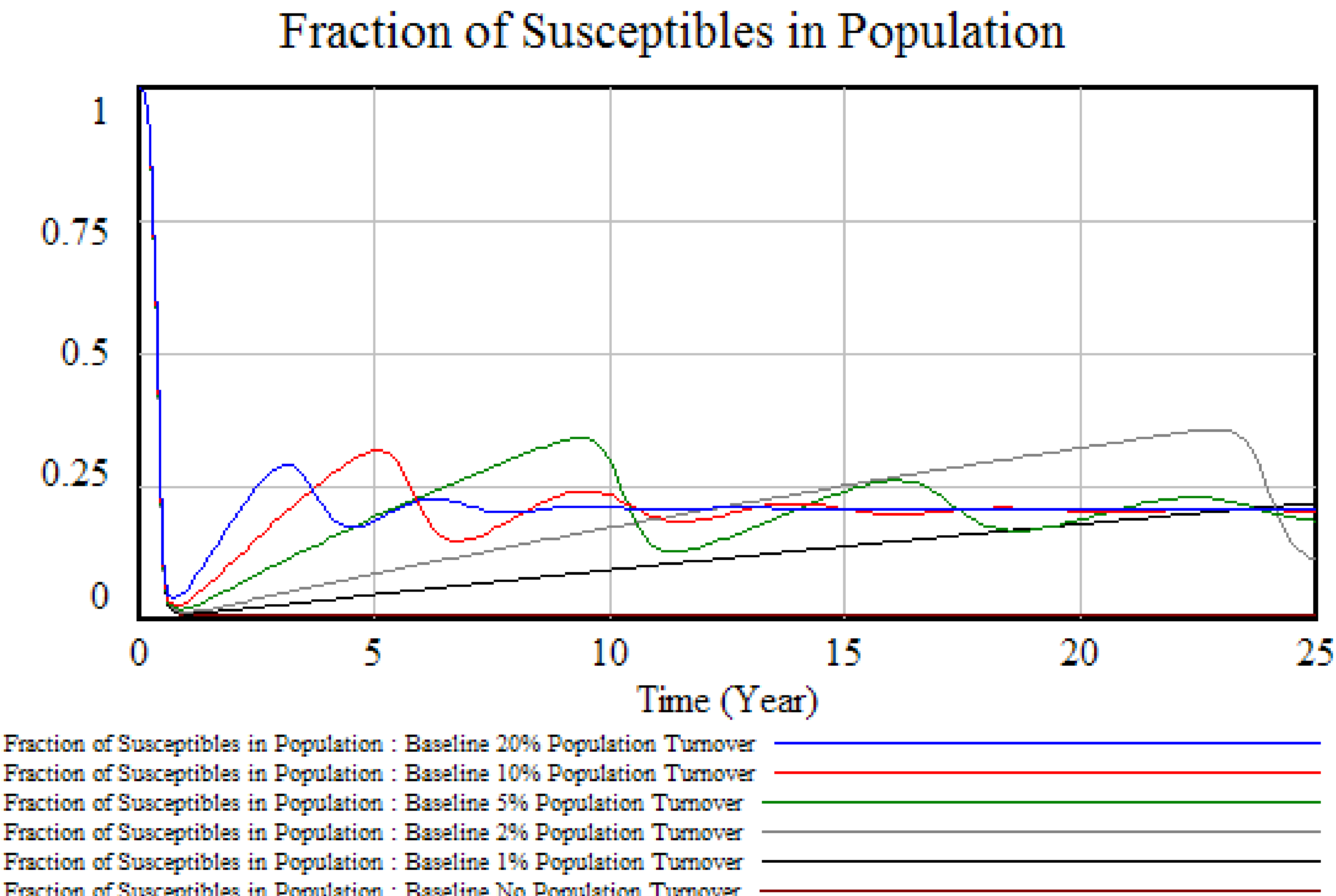


Recovered
1,000
750
500
250
0
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800
850
900
950
100
Time (Month)
Recovered : Baseline 2% Annual Turnover
Recovered : Baseline Closed Population

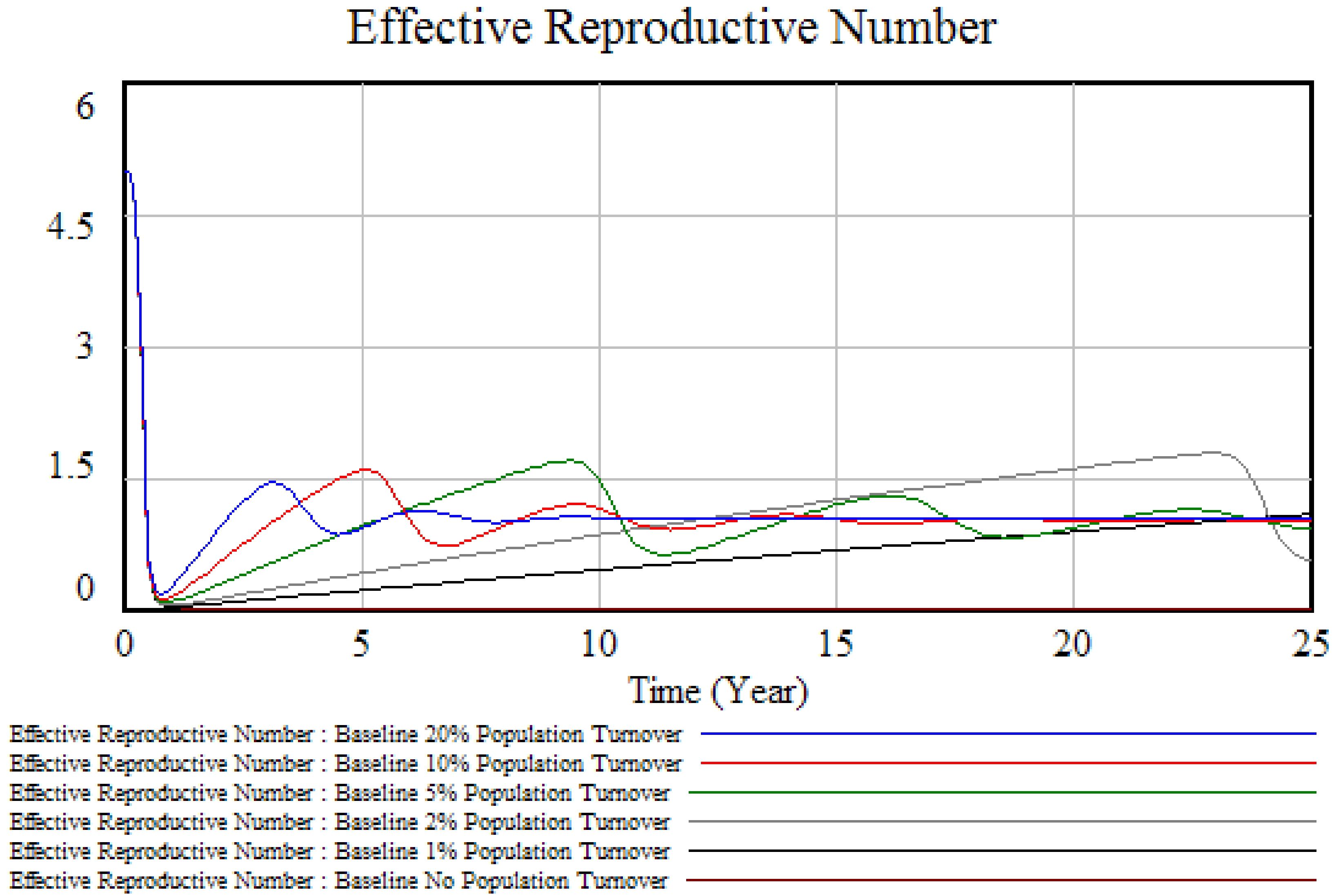
Impact of Turnover

- The greater the turnover rate, the greater the fraction of susceptibles in the population => the greater the endemic rate of infection

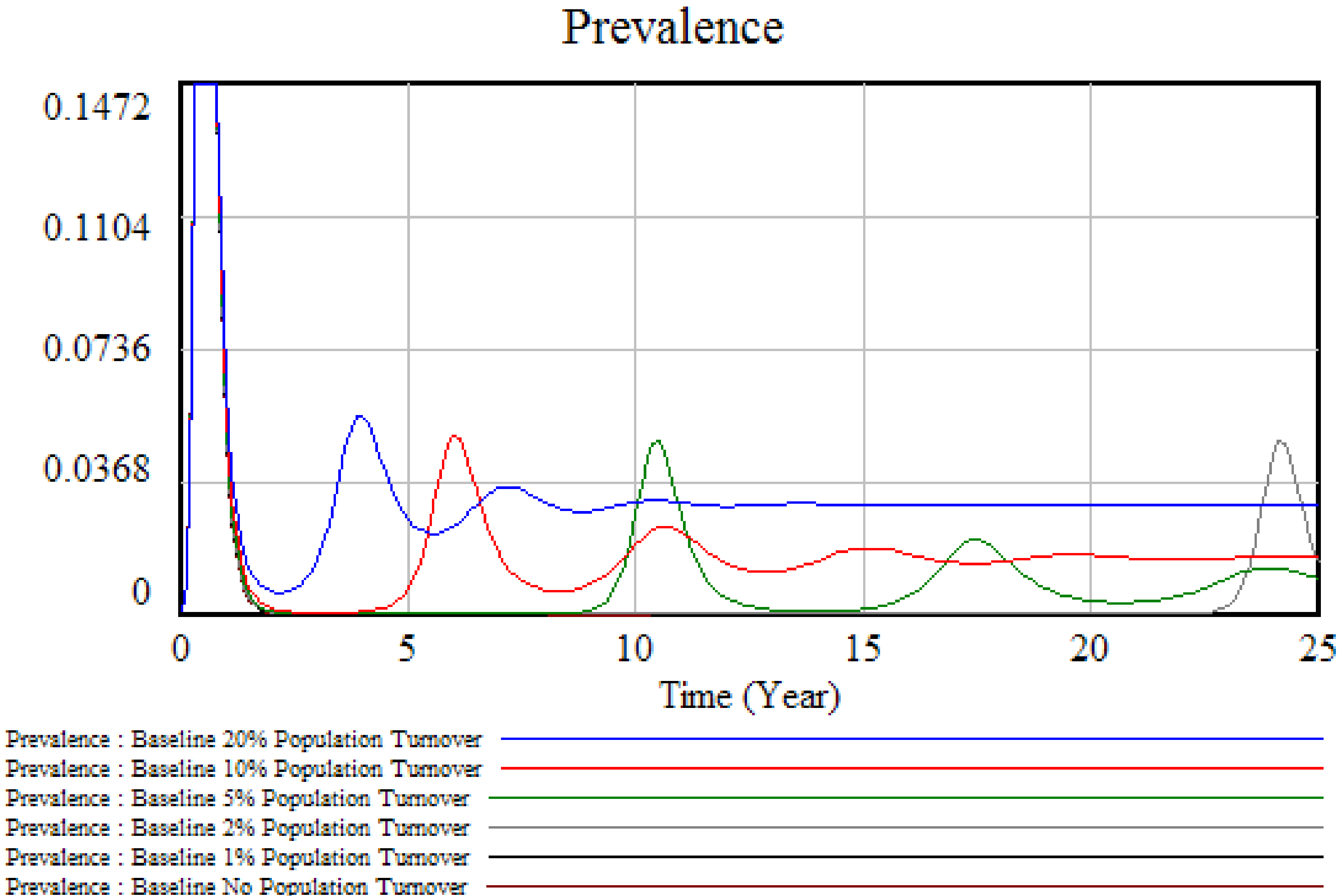
Fraction of Susceptibles



Effective Reproductive Number

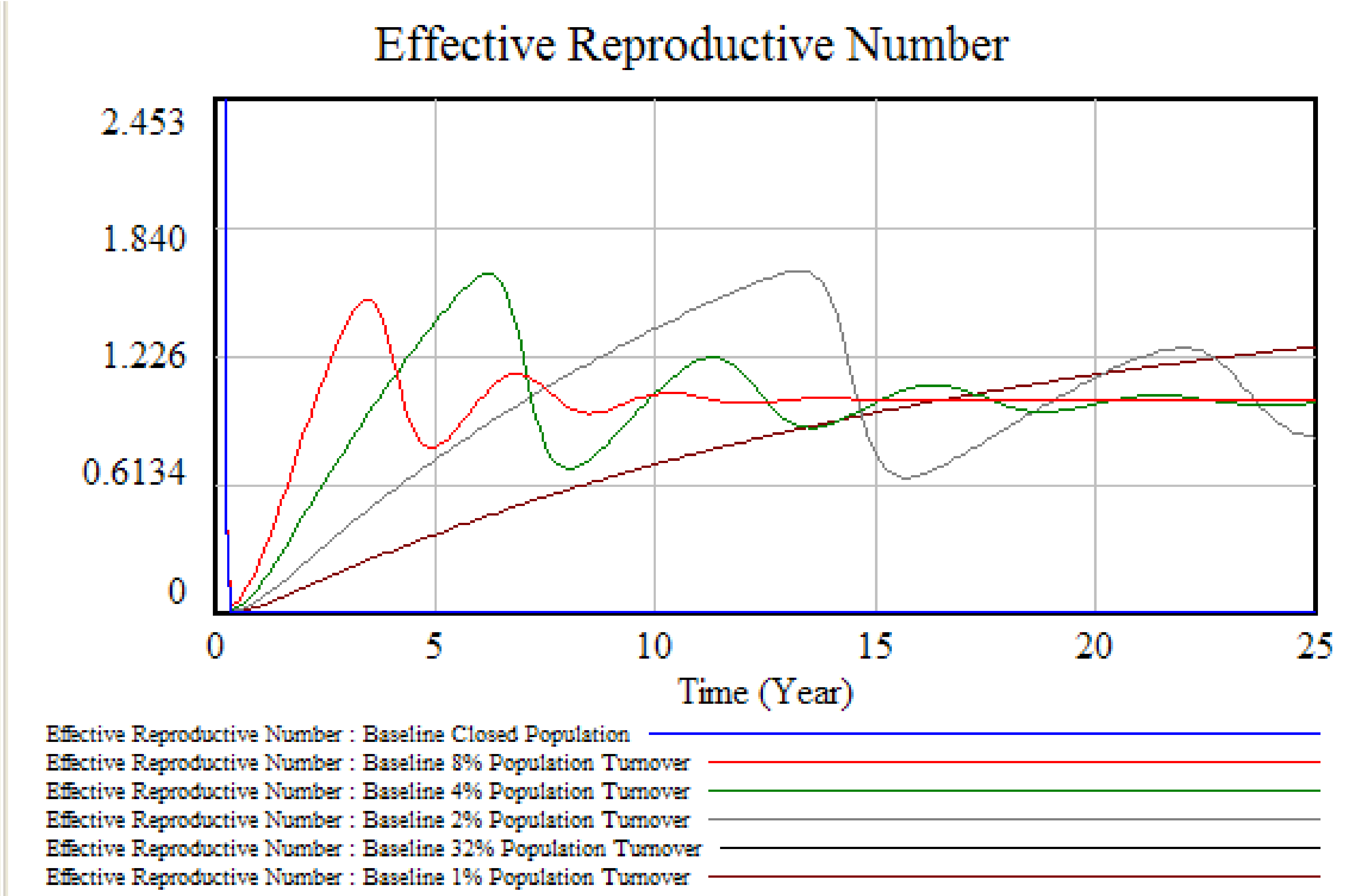


Prevalence



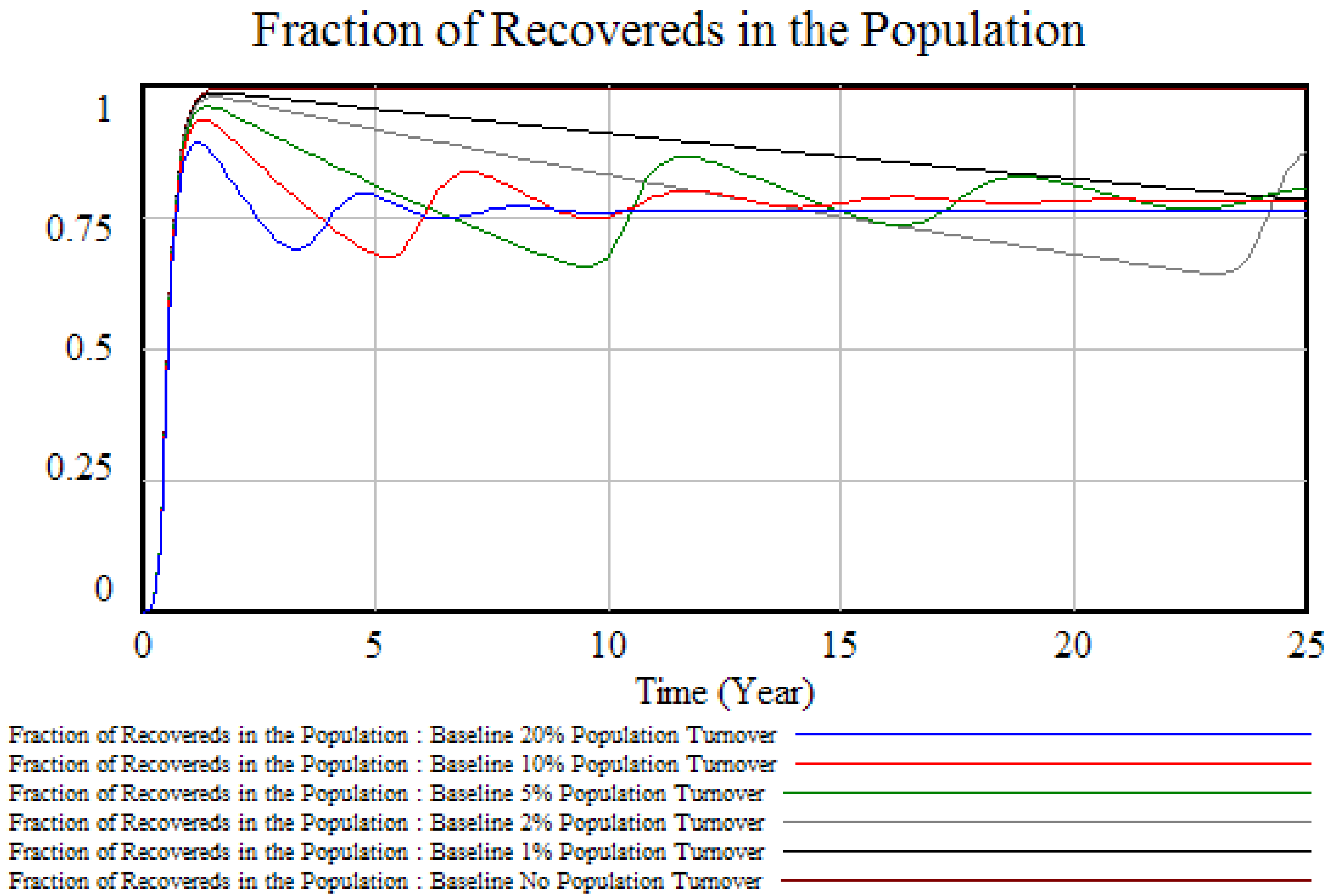
Prevalence
0.1472
0.1104
0.0736
0.0368
0
0
5
10
15
20
25
Time (Year)
Prevalence : Baseline 20% Population Turnover
Prevalence : Baseline 10% Population Turnover
Prevalence : Baseline 5% Population Turnover
Prevalence : Baseline 2% Population Turnover
Prevalence : Baseline 1% Population Turnover
Prevalence : Baseline No Population Turnover

R_*

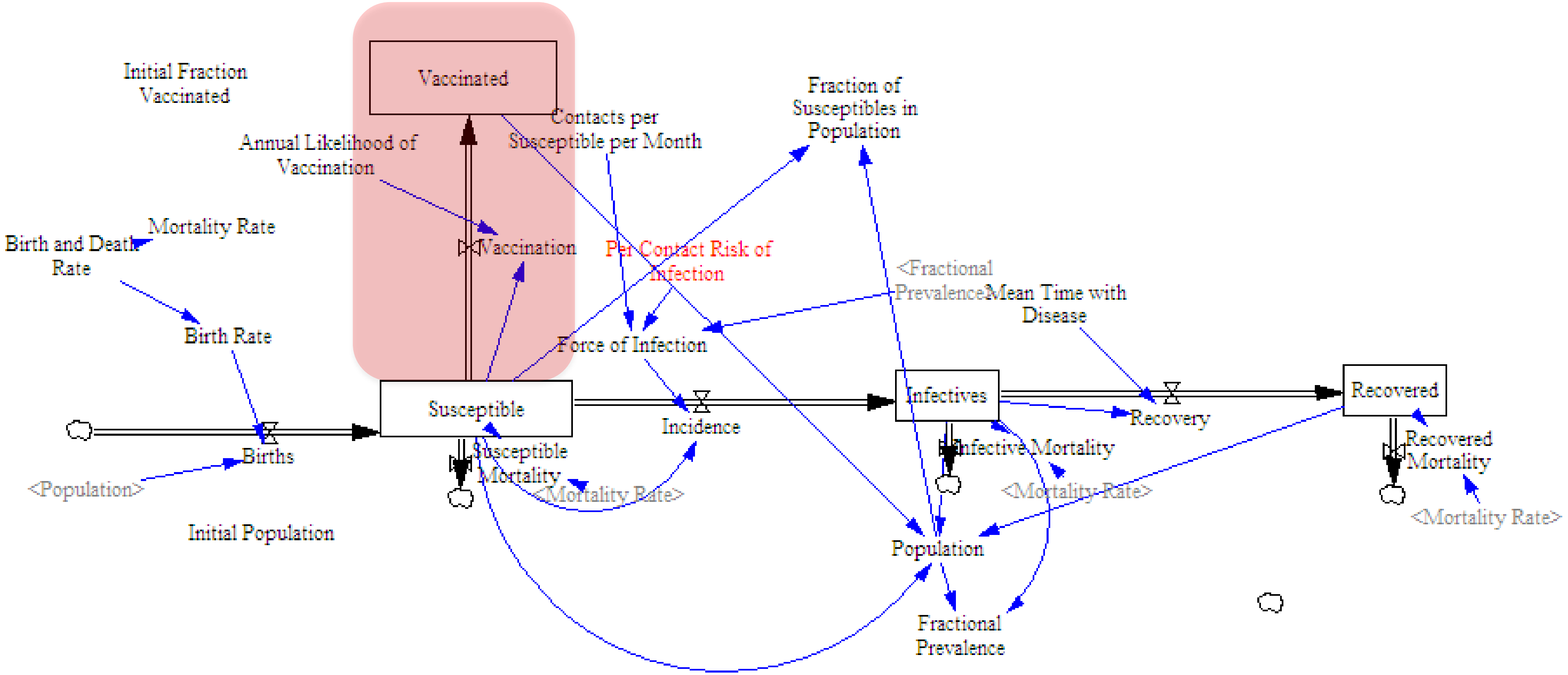


Effective Reproductive Number
2.453
1.840
1.226
0.6134
0
0
5
10
15
20
25
Time (Year)
Effective Reproductive Number : Baseline Closed Population
Effective Reproductive Number : Baseline 8% Population Turnover
Effective Reproductive Number : Baseline 4% Population Turnover
Effective Reproductive Number : Baseline 2% Population Turnover
Effective Reproductive Number : Baseline 32% Population Turnover
Effective Reproductive Number : Baseline 1% Population Turnover

Fraction Recovered

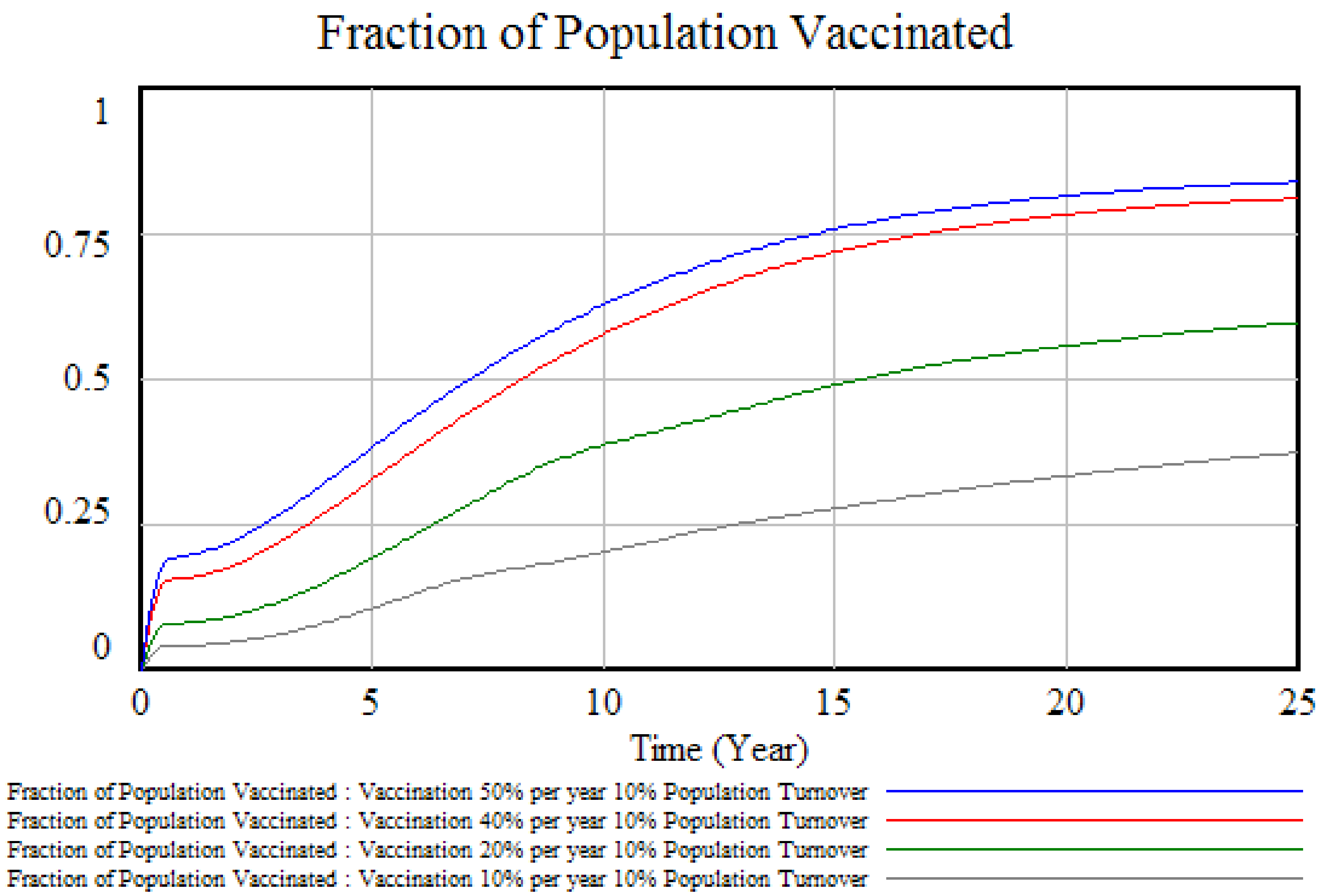


Adding Ongoing Vaccination Process

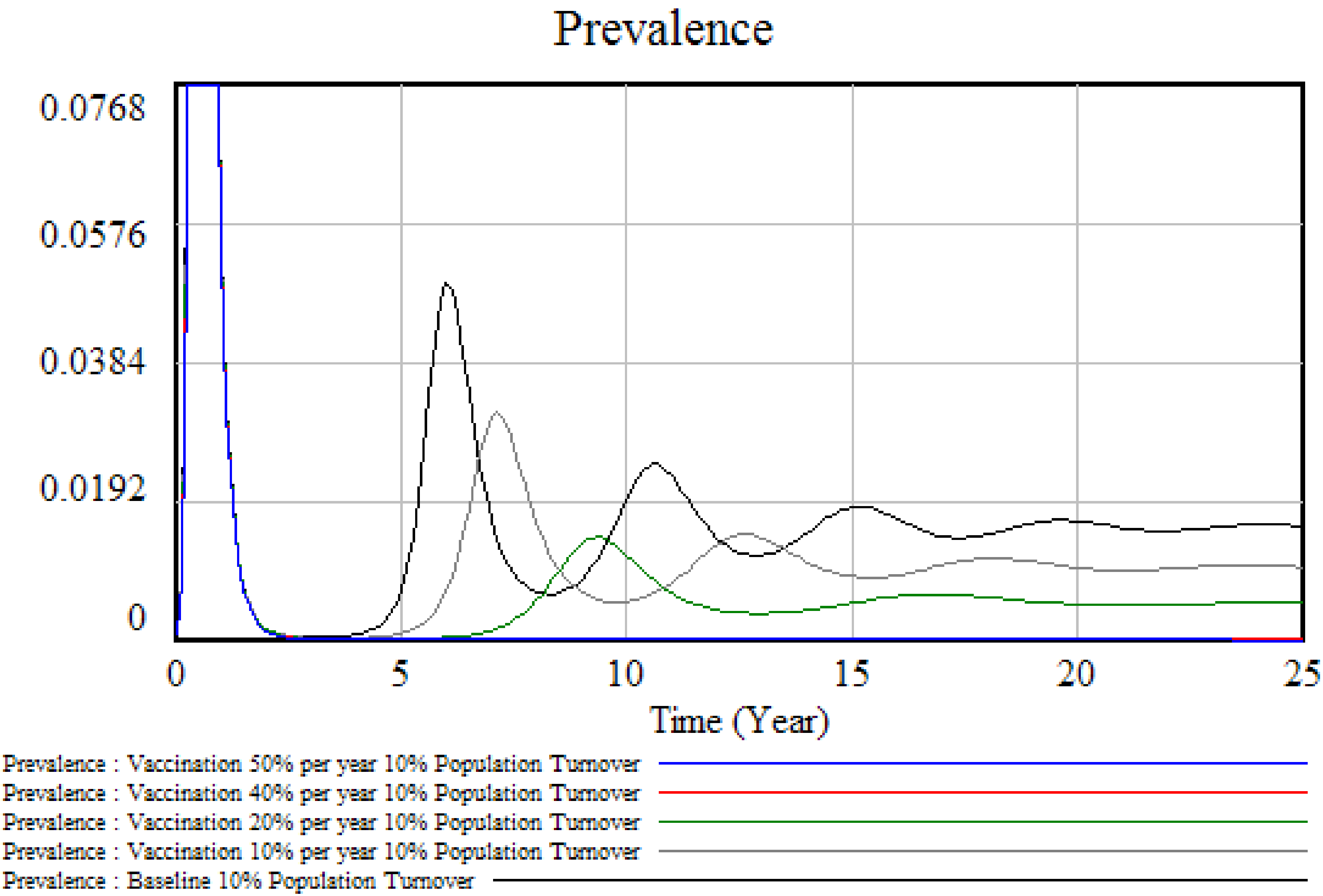


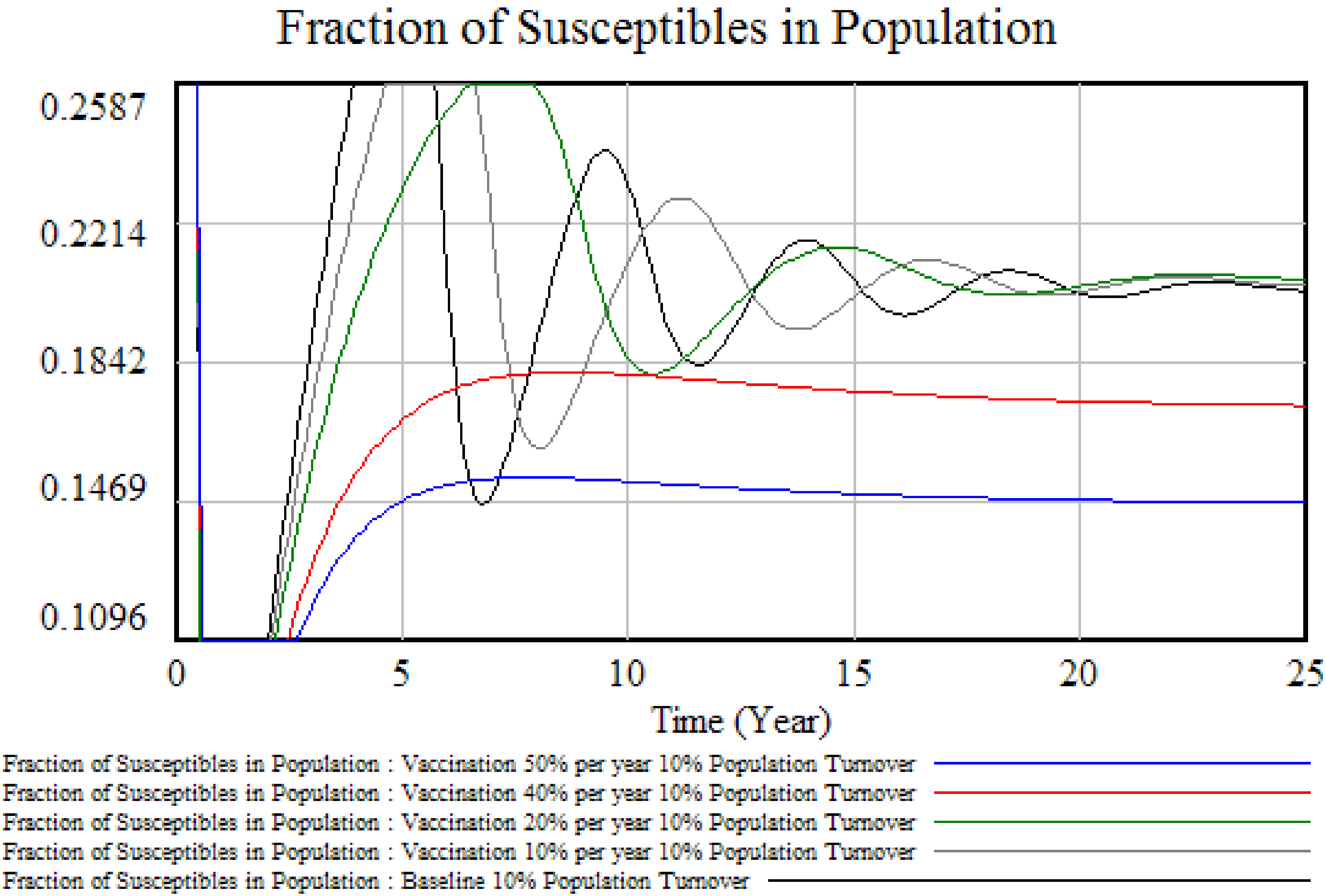
Simulating Introduction of Vaccination for a Childhood Infection in an Open Population

- c = 500
- Beta = 0.05
- Duration of infection = .25
- Initial Fraction Vaccinated = 0
- Monthly birth & death rate = 10% per year
  (focusing on children 0-10 years of age)
- Questions
  - What is R_0?
  - What level of susceptibles is required to sustain the infection
  - What is the critical vaccination fraction?

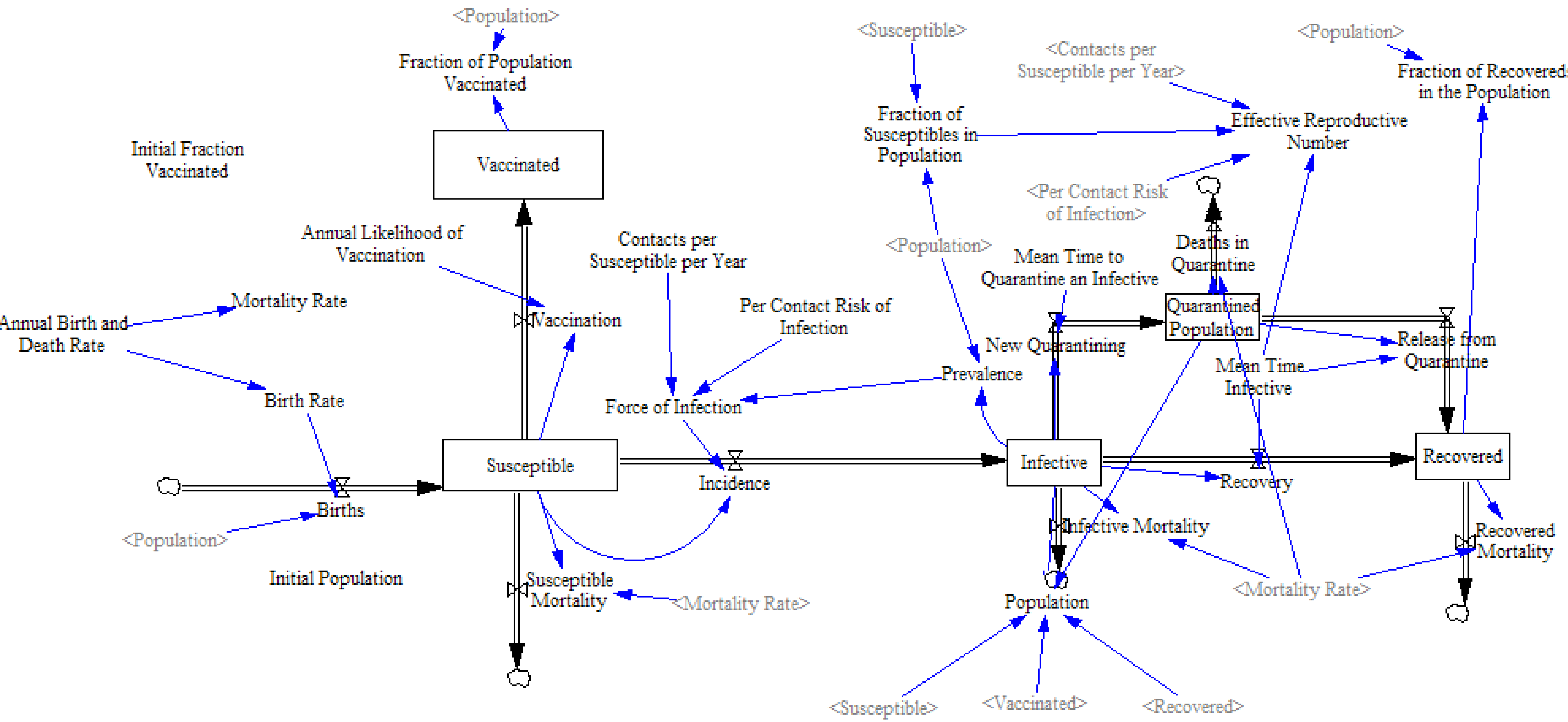

Fraction of Population Vaccinated
1
0.75
0.5
0.25
0
0
5
10
15
20
25
Time (Year)
Fraction of Population Vaccinated : Vaccination 50% per year 10% Population Turnover
Fraction of Population Vaccinated : Vaccination 40% per year 10% Population Turnover
Fraction of Population Vaccinated : Vaccination 20% per year 10% Population Turnover
Fraction of Population Vaccinated : Vaccination 10% per year 10% Population Turnover

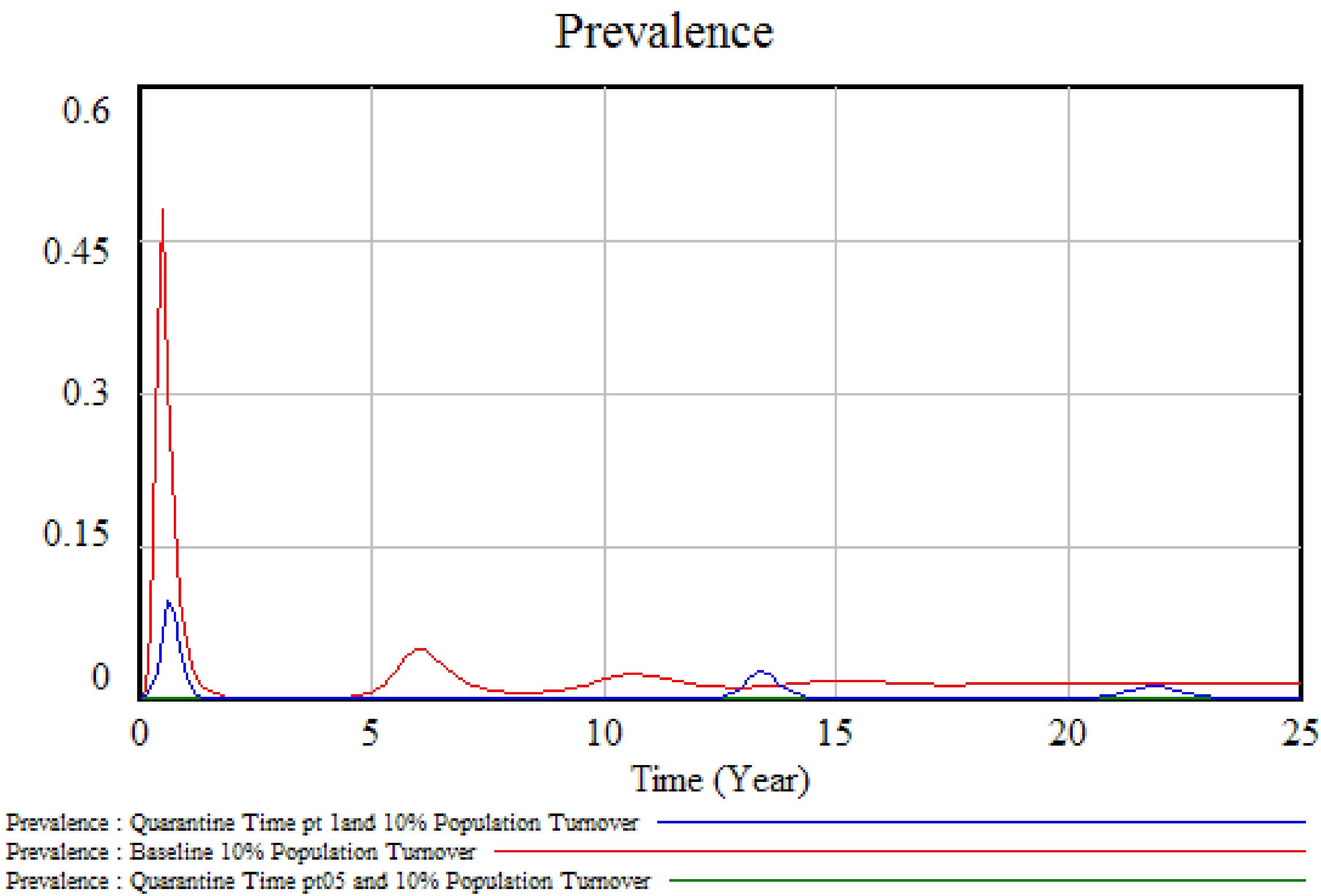
What Rate of Vaccination Eliminates?

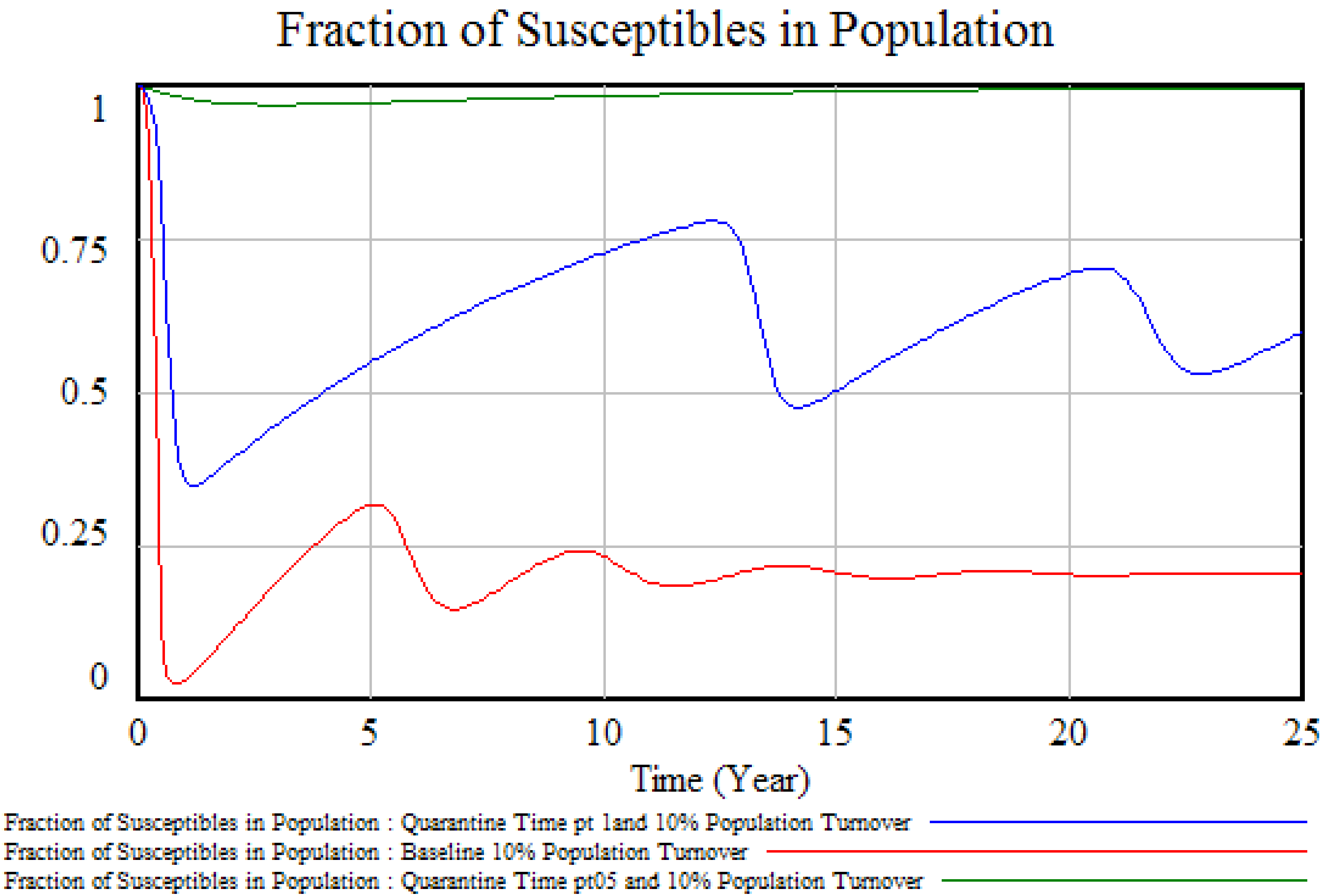



Fraction of Susceptibles in Population
0.2587
0.2214
0.1842
0.1469
0.1096
0
5
10
15
20
25
Time (Year)
Fraction of Susceptibles in Population : Vaccination 50% per year 10% Population Turnover
Fraction of Susceptibles in Population : Vaccination 40% per year 10% Population Turnover
Fraction of Susceptibles in Population : Vaccination 20% per year 10% Population Turnover
Fraction of Susceptibles in Population : Vaccination 10% per year 10% Population Turnover
Fraction of Susceptibles in Population : Baseline 10% Population Turnover

Representing Quarantine




Prevalence
0.6
0.45
0.3
0.15
0
0
5
10
15
20
25
Time (Year)
Prevalence : Quarantine Time pt 1and 10% Population Turnover
Prevalence : Baseline 10% Population Turnover
Prevalence : Quarantine Time pt05 and 10% Population Turnover


Fraction of Susceptibles in Population
1
0.75
0.5
0.25
0
0
5
10
15
20
25
Time (Year)
Fraction of Susceptibles in Population : Quarantine Time pt 1and 10% Population Turnover
Fraction of Susceptibles in Population : Baseline 10% Population Turnover
Fraction of Susceptibles in Population : Quarantine Time pt05 and 10% Population Turnover

Endemic Situations

- In an endemic context, infection remains circulating in the population
- The common assumption here is that
  - The susceptible portion of the population will be children
  - At some point in their life trajectory (at an average age of acquiring infection A), individuals will be exposed to the infection & develop immunity

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Age of Exposure & Reproductive Constant

- Cf a "natural" (non-immunized) constant size population where all die at same age and where
  - Mean Age at death L
  - Mean Age of exposure A (i.e. we assume those above A are exposed)
- Fraction susceptible is S/N = A/L (i.e. proportion of population below age A)
- Recall for our (and many but not *all* other) models: $R^*=(S/N)R_0=1 \Rightarrow S/N=1/R_0$
- Thus

$$A/L = 1/R_0 \Rightarrow L/A = R_0$$

  - This tells us that the larger the R_0, the earlier in life individuals become infected

Incompletely Immunized Population

- Suppose we have *q* fraction of population immunized ($q<q_c$)
- Suppose we have fraction *f* susceptible
- Fraction of the population currently or previously *infected* is 1-q-f
  - If we assume (as previously) that everyone lives until L and is infected at age A, then fraction 1-A/L has been infected
  - So $1-A/L= 1-q-f \Rightarrow A = L(q+f)$
    - This can be much higher than for the natural population
      - This higher age of infection can cause major problems, due to waning of childhood defenses
    - i.e. incomplete immunization leads to older mean age ofexposure

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